Chapter 7: Proportions and Percents

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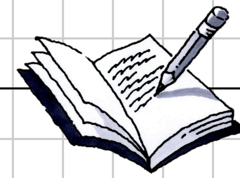

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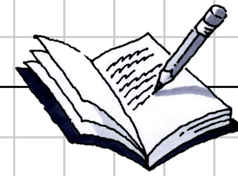

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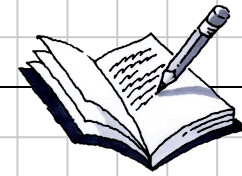
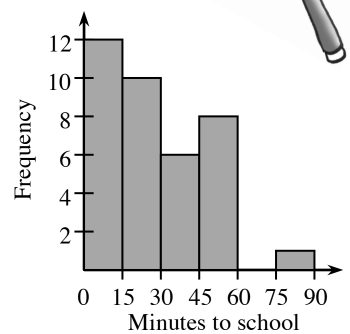


Notes:

MATH NOTES

HISTOGRAMS AND STEM-AND-LEAF PLOTS

A **histogram** is similar to a bar graph in that each bar represents data in an interval of numbers. The intervals for the data are shown on the horizontal axis, and the frequency (number of pieces of data in each interval) is represented by the height of a bar above the interval.



The labels on the horizontal axis represent the lower end of each interval. For example, the histogram at right shows that 10 students take at least 15 minutes but less than 30 minutes to get to school.

Histograms are used to display numeric data with an order, while bar graphs display data in categories where order generally does not matter.

A **stem-and-leaf plot** shows the same data as a histogram, but it shows the individual values from a set of data and how the values are distributed. The "stem" part of the graph represents all of the digits in a number except the last one. The "leaf" part of the graph represents the last digit of each of the numbers.

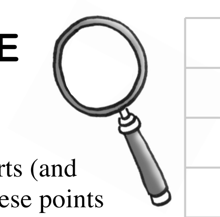
Example: Students in a math class received the following scores on their tests: 49, 52, 54, 58, 61, 61, 67, 68, 72, 73, 73, 73, 78, 82, and 83. Display the test-score data on a stem-and-leaf plot.

"leaves"

"stem"

4	9				
5	2	4	8		
6	1	1	7	8	
7	2	3	3	3	8
8	2	3			

QUARTILES AND INTERQUARTILE RANGE (IQR)



Quartiles are points that divide a data set into four equal parts (and thus, the use of the prefix "quar" as in "quarter"). One of these points is the median, which you learned about in Chapter 1, since it marks the middle of the data set. In addition, there are two other quartiles in the middle of the lower and upper halves: the **first quartile** and the **third quartile**.

Suppose you have this data set: 22, 43, 14, 7, 2, 32, 9, 36, and 12.

To find quartiles, the data set must be placed in order from smallest to largest. Then divide the data set into two halves by finding the median of the entire data set. Next, find the median of the lower and upper halves of the data set. (Note that if there is an odd number of data values, the median is not included in either half of the data set.) See the example below.

2 7 9 12 | 14 | 22 32 36 43

lower half: 2 7 9 12 → 8 **first quartile** (median of the lower half)

14 → median

upper half: 22 32 36 43 → 34 **third quartile** (median of the upper half)

The **interquartile range** (IQR) is one way (along with range) to measure the spread of the data. Statisticians often prefer using the IQR to measure spread because it is not affected much by outliers or non-symmetrical distributions. The IQR is the range of the middle 50% of the data. It is calculated by subtracting the first quartile from the third quartile. In the above example, the IQR is 26 (34 – 8 = 26).

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DESCRIBING DATA DISTRIBUTIONS

Distributions of data are typically described by considering the **center**, **shape**, **spread**, and **outliers**.

Center: The median best represents the center (or a "typical" data value) if the distribution is not symmetrical or if there are outliers. Either the mean or the median is appropriate for describing the center of symmetrical distributions with no outliers.

Shape: The shape is the overall appearance of the data when it is displayed in a histogram or stem-and-leaf plot. Is the distribution fairly symmetrical? Uniform? Single peaked? Skewed? Does it have large gaps or clusters?

Spread: Spread is a measure of the variability of the data, that is, how much scatter there is in the data. For non-symmetrical data or data with outliers, use the interquartile range (IQR) to describe the spread, since it is based on median. For symmetrical data with no outliers, either the mean absolute deviation, which is based on the mean, or the IQR are appropriate measures of spread. The range is not usually the best measure of the scatter in data, because it considers only the maximum and the minimum values and not what is occurring in between.

Outliers: An outlier is any data point that is far removed from the bulk of the rest of the data.

SCALING

When a quantity is increased or decreased by a specific proportion of the original amount, it is changed by a specific scale factor (also called a multiplier). Quantities are **scaled up** when they are increased by multiplying by a number greater than one or **scaled down** when they are decreased by multiplying by a number between (but not including) zero and one.

For example, if a music system is on sale for 25% off its original price of $500, the discount can be found by multiplying by 25%:

$$\text{discount} = 0.25(\text{original price}) = 0.25(\$500) = \$125$$

The full price (100%) minus the discount (25%) would result in the sale price, which in this case is 75% of the original. The sale price can also be found by scaling:

$$\text{sale price} = 0.75(\text{original price}) = 0.75(\$500) = \$375$$

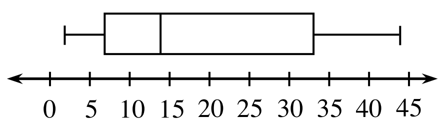
Scaling can be used to enlarge and reduce side lengths of similar shapes, or to increase or decrease times, distances, and other related quantities.

Notes:

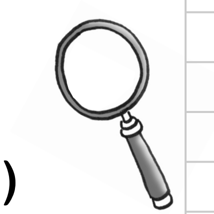
BOX PLOTS

A **box plot** (also known as a "box-and-whiskers" plot) displays a summary of data using the median, quartiles, and extremes of the data. The box contains "the middle half" of the data. The right segment represents the top 25% of the data, and the left segment represents the bottom 25% of the data. A box plot makes it easy to see where the data are spread out and where they are concentrated. The larger the box, the more the data are spread out.

To construct a box plot using a number line that shows the range of the data, draw vertical line segments above the median, first quartile and third quartile. Then connect the lines from the first and third quartiles to form a rectangle. Place a vertical line segment above the number line at the maximum (highest) and minimum (lowest) data values. Connect the minimum value to the first quartile and the maximum value to the third quartile using horizontal segments. The box plot is shown below for the data set 2, 7, 9, 12, 14, 22, 32, 36, and 43.



SOLVING EQUATIONS WITH ALGEBRAIC FRACTIONS (ALSO KNOWN AS FRACTION BUSTERS)



Example: Solve $\frac{x}{3}+\frac{x}{5}=2$ for x.

$$\frac{x}{3}+\frac{x}{5}=2$$

This equation would be much easier to solve if it had no fractions. Therefore, the first goal is to find an equivalent equation that has no fractions.

The lowest common denominator of $\frac{x}{3}$ and $\frac{x}{5}$ is 15.

To eliminate the denominators, multiply all of the terms on both sides of the equation by the common denominator. In this example, the lowest common denominator is 15, so multiplying all of the terms (both sides) in the equation by 15 eliminates the fractions. Another approach is to multiply all of the terms in the equation by one denominator and then by the other. Either way, the result is an equivalent equation without fractions.

$$15\cdot\left(\frac{x}{3}+\frac{x}{5}\right)=15\cdot 2$$

$$15\cdot\frac{x}{3}+15\cdot\frac{x}{5}=15\cdot 2$$

$$5x+3x=30$$

$$8x=30$$

In this course, the number used to eliminate the denominators is called a **Fraction Buster**. Now the equation looks like many you have seen before, and it can be solved in the usual way.

$$x=\frac{30}{8}=\frac{15}{4}=3.75$$

Check: $\frac{3.75}{3}+\frac{3.75}{5}=2$

$1.25+0.75=2$ ✓

Once you have found the solution, remember to check your answer.

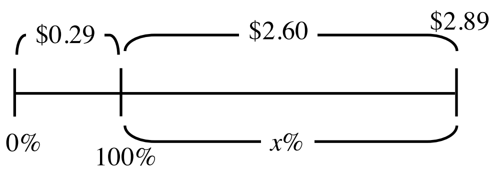
Notes:

Percent Increase or Decrease

A **percent increase** is the amount that a quantity has increased, represented as a percent of the original amount. A **percent decrease** is the amount that a quantity has decreased, written as a percent of the original amount. You can write an equation to represent a percent change that is an increase or decrease using a scale factor or multiplier:

amount of increase or decrease = (% change)(original amount)

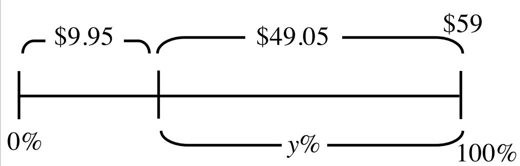
Example 1: A loaf of bread increased in price from \$0.29 to \$2.89 in the past 50 years. What was the percent increase?



$$\text{increase} = \$2.89 - \$0.29$$
$$= \$2.60$$

$$\$2.60 = (x)(\$0.29)$$
$$\frac{\$2.60}{\$0.29} = x$$
$$x \approx 8.97 \text{ or } 897\%$$

Example 2: Calculator prices decreased from \$59 to \$9.95. What was the percent decrease?



$$\text{decrease} = \$59 - \$9.95$$
$$= \$49.05$$

$$\$49.05 = (y)(\$59)$$
$$\frac{\$49.05}{\$59} = y$$
$$y \approx 0.83 = 83\%$$

Simple Interest

Simple interest is interest paid only on the original amount of the principal at each specified interval (such as annually or monthly). The formula to calculate simple interest is:

$I = Prt$ where P = Principal, I = Interest, r = Rate, t = Time

Example: Theresa invested \$1425.00 in a savings account at her local bank. The bank pays a simple interest rate of 3.5% annually. How much money will Theresa have after 4 years?

$$I = Prt \quad \Rightarrow \quad I = 1425(0.035)(4) = \$199.50$$
$$\Rightarrow \quad P + I = \$1425 + \$199.50 = \$1624.50$$

Theresa will have \$1624.50 after 4 years.

SOLVING PROPORTIONS

Notes:

An equation stating that two ratios are equal is called a **proportion**. Some examples of proportions are shown at right.

$$\frac{6\text{ mi}}{2\text{ hr}} = \frac{9\text{ mi}}{3\text{ hr}}$$

$$\frac{5}{7} = \frac{50}{70}$$

When two ratios are known to be equal, setting up a proportion is one strategy for solving for an unknown part of one ratio. For example, if the ratios $\frac{9}{2}$ and $\frac{x}{16}$ are equal, setting up the proportion $\frac{x}{16} = \frac{9}{2}$ allows you to solve for x.

Strategy 1: One way to solve this proportion is by using a **Giant One** to find the equivalent ratio. In this case, since 2 times 8 is 16, so use $\frac{8}{8}$ for the Giant One.

$\frac{x}{16} = \frac{9}{2} \cdot \frac{8}{8}$ and $\frac{9 \cdot 8}{2 \cdot 8} = \frac{72}{16}$, which shows that $\frac{x}{16} = \frac{72}{16}$, so $x = 72$.

Strategy 2: Undoing division. Another way to solve the proportion is to think of the ratio $\frac{x}{16}$ as, "x divided by 16." To solve for x, use the inverse operation of division, which is multiplication. Multiplying both sides of the proportional equation by 16 "undoes" the division.

$$\frac{x}{16} = \frac{9}{2}$$

$$\left(\frac{16}{1}\right)\frac{x}{16} = \frac{9}{2}\left(\frac{16}{1}\right)$$

$$x = \frac{144}{2} = 72$$

Strategy 3: Use cross multiplication. This is a solving strategy for proportions that is based on the process of multiplying each side of the equation by the denominators of each ratio and setting the two sides equal. It is a shortcut for using a **Fraction Buster** (multiplying each side of the equation by the denominators).

Complete Algebraic Solution (Fraction Busters)

$$\frac{x}{16} = \frac{9}{2}$$

$$2 \cdot 16 \cdot \frac{x}{16} = \frac{9}{2} \cdot 2 \cdot 16$$

$$2 \cdot x = 9 \cdot 16$$

$$2x = 144$$

$$x = 72$$

Cross Multiplication

$$\frac{x}{16} = \frac{9}{2}$$

$$\frac{x}{16} \times \frac{9}{2}$$

$$2 \cdot x = 9 \cdot 16$$

$$2x = 144$$

$$x = 72$$