


CHAPTER 7: RATES AND OPERATIONS

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MATH NOTES

RATES AND UNIT RATES

In Lesson 7.1.1, you learned that a **rate** is a ratio that compares two different quantities.

$$\text{rate} = \frac{\text{one quantity}}{\text{another quantity}}$$



A **unit rate** is a rate that compares the change in one quantity to a 1-unit change in another quantity. For example, *miles per hour* is a unit rate, because it compares the change in miles to a change of 1 hour. If an airplane flies 3000 miles in 5 hours and uses 6000 gallons of fuel, you can compute several unit rates.

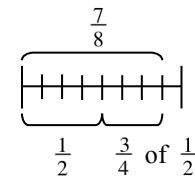
It uses $\frac{6000 \text{ gallons}}{5 \text{ hours}} = 1200 \frac{\text{gallons}}{\text{hour}}$ or $\frac{6000 \text{ gallons}}{3000 \text{ miles}} = 2 \frac{\text{gallons}}{\text{mile}}$, and it travels at $\frac{3000 \text{ miles}}{5 \text{ hours}} = 600 \frac{\text{miles}}{\text{hour}}$.

FRACTION DIVISION, PART 1

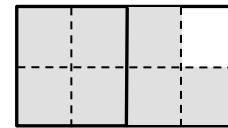
Method 1: Using Diagrams

To divide by a fraction using a diagram, create a model of the situation using rectangles, a linear model, or another visual representation of it. Then break that model into the fractional parts named.

For example, to divide $\frac{7}{8} \div \frac{1}{2}$, you can draw the diagram at right to visualize how many $\frac{1}{2}$ -sized pieces fit into $\frac{7}{8}$. The diagram shows that one $\frac{1}{2}$ fits one time, with $\frac{3}{8}$ of a whole left. Since $\frac{3}{8}$ is $\frac{3}{4}$ of $\frac{1}{2}$, you can see that $1 \frac{3}{4}$ $\frac{1}{2}$ -sized pieces fit into $\frac{7}{8}$, so $\frac{7}{8} \div \frac{1}{2} = 1 \frac{3}{4}$.



Alternately, you could think of $\frac{7}{8}$ as the quantity that you have and $\frac{1}{2}$ as the size of the group that you want, such as having $\frac{7}{8}$ ounce of chocolate and needing $\frac{1}{2}$ ounce for each cake recipe. How many cakes could you make? In this case the diagram at right might be useful. The diagram shows $\frac{7}{8}$ being divided into groups of $\frac{1}{2}$. The leftover $\frac{3}{8}$ ounce creates another $\frac{3}{4}$ of a group, so again, $\frac{7}{8} \div \frac{1}{2} = 1 \frac{3}{4}$.



Method 2: Using Common Denominators

To divide a number by a fraction using common denominators, express both numbers as fractions with the same denominator. Then divide the first numerator by the second. An example is shown at right.

$$\begin{aligned} \frac{2}{5} \div \frac{3}{10} &= \frac{4}{10} \div \frac{3}{10} \\ &= 4 \div 3 \\ &= \frac{4}{3} = 1 \frac{1}{3} \end{aligned}$$

MULTIPLICATIVE INVERSES AND RECIPROCALLS



Two numbers with a product of 1 are called **multiplicative inverses**.

$$\frac{8}{5} \cdot \frac{5}{8} = \frac{40}{40} = 1 \quad 3 \frac{1}{4} = \frac{13}{4}, \text{ so } 3 \frac{1}{4} \cdot \frac{4}{13} = \frac{13}{4} \cdot \frac{4}{13} = \frac{52}{52} = 1 \quad \frac{1}{7} \cdot 7 = 1$$

In general $a \cdot \frac{1}{a} = 1$ and $\frac{a}{b} \cdot \frac{b}{a} = 1$, where neither a nor b equals zero. Note that $\frac{1}{a}$ is the **reciprocal** of a and $\frac{b}{a}$ is the reciprocal of $\frac{a}{b}$. Also note that 0 has no reciprocal.

FRACTION DIVISION, PART 2



Method 3: Using a Super Giant One

To divide by a fraction using a Super Giant One, write the two numbers (dividend and divisor) as a complex fraction with the dividend as the numerator and the divisor as the denominator. Use the reciprocal of the complex fraction's denominator to create a Super Giant One. Then simplify, as shown in the following examples.

$$6 \div \frac{3}{4} = \frac{6}{\frac{3}{4}} = \frac{6}{\frac{3}{4}} \cdot \frac{\frac{4}{3}}{\frac{4}{3}} = \frac{6 \cdot 4}{1 \cdot 3} = \frac{6}{1} \cdot \frac{4}{3} = \frac{24}{3} = 8$$

$$\frac{3}{4} \div \frac{2}{5} = \frac{\frac{3}{4}}{\frac{2}{5}} = \frac{\frac{3}{4}}{\frac{2}{5}} \cdot \frac{\frac{5}{2}}{\frac{5}{2}} = \frac{3 \cdot 5}{4 \cdot 2} = \frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8} = 1 \frac{7}{8}$$

Method 4: Using the “Invert and Multiply” Method

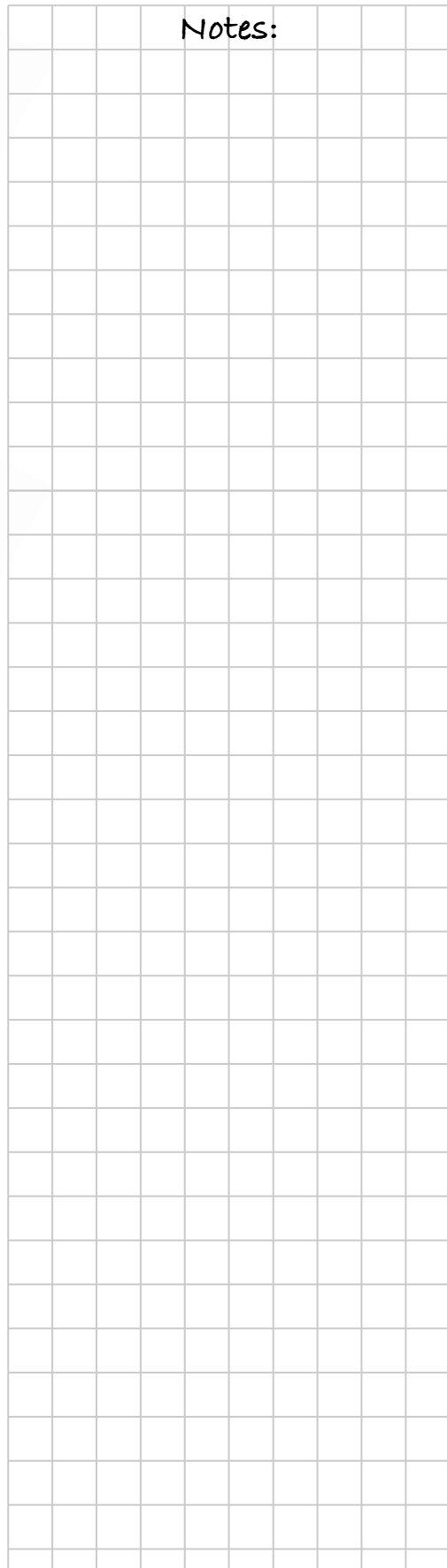
Notice that the result of multiplying by the Super Giant One in the above examples is that the denominator of the complex fraction is always 1. The resulting numerator is the product of the first fraction (dividend) and the reciprocal of the second fraction (divisor).

To use the “Invert and Multiply” method, multiply the first fraction (dividend) by the reciprocal (multiplicative inverse) of the second fraction (divisor). If the first number is an integer, write it as a fraction over 1.

Here is the second problem from the examples above that was solved with the Invert and Multiply method:

$$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8} = 1 \frac{7}{8}$$

Notes:



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DISTRIBUTIVE PROPERTY WITH VARIABLES



Remember that the **Distributive Property** states that multiplication can be “distributed” as a multiplier of each term in a sum or difference. Symbolically, this can be written as:

$$a(b + c) = ab + ac \quad \text{and} \quad a(b - c) = ab - ac$$

For example, the collection of tiles at right can be represented as 4 sets of $x + 3$, written as $4(x + 3)$. It can also be represented by 4 x -tiles and 12 unit tiles, written as $4x + 12$.

$$4(x + 3) = 4x + 12$$

MATHEMATICS VOCABULARY



Variable: A letter or symbol that represents one or more numbers.

Expression: A combination of numbers, variables, and operation symbols. For example, $2x + 3(5 - 2x) + 8$. Also, $5 - 2x$ is a smaller expression within the larger expression.

Term: Parts of the expression separated by addition and subtraction. For example, in the expression $2x + 3(5 - 2x) + 8$, the three terms are $2x$, $3(5 - 2x)$, and 8 . The expression $5 - 2x$ has two terms, 5 and $2x$.

Coefficient: The numerical part of a term. In the expression $2x + 3(5 - 2x) + 8$, for example, 2 is the coefficient of $2x$. In the expression $7x - 15x^2$, both 7 and 15 are coefficients.

Constant term: A number that is not multiplied by a variable. In the expression $2x + 3(5 - 2x) + 8$, the number 8 is a constant term. The number 3 is not a constant term, because it is multiplied by a variable inside the parentheses.

Factor: Part of a multiplication expression. In the expression $3(5 - 2x)$, 3 and $5 - 2x$ are factors.

SOLVING AND GRAPHING INEQUALITIES



An **equation** always has an equal sign. An **inequality** has a mathematical inequality (comparison) symbol in it. To **solve** an equation or inequality means to find all the values of the variable that make the equation true. See the examples below.

Solve this equation:

$$x + 3 = 7$$

The solution is:

$$x = 4$$

Solve this inequality:

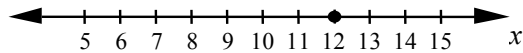
$$x - 2 < 5$$

The solution is:

$$x < 7$$

To solve and graph an inequality with one variable, first treat the problem as if it were an equality and solve the problem. The solution to the equality is called the **boundary point**. For example, to solve $x - 4 \geq 8$, solve $x - 4 = 8$. The solution $x = 12$ is the boundary point for the inequality $x - 4 \geq 8$.

Since the original inequality is true when $x = 12$, place your boundary point on the number line as a solid point. Then test one value on either side in the *original* inequality to determine which set of numbers makes the inequality true. This is shown with the examples of $x = 8$ and $x = 15$ at right. After testing you can see that the solution is $x \geq 12$.



Test : $x = 8$

$$8 - 4 \geq 8$$

$$4 \geq 8$$

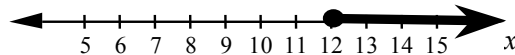
FALSE!

Test : $x = 15$

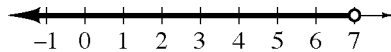
$$15 - 4 \geq 8$$

$$11 \geq 8$$

TRUE!



When the inequality is $<$ or $>$, the boundary point is *not* included in the answer. On a number line, this would be indicated with an open circle at the boundary point. For example, the graph of $x < 7$ is shown below.



Notes: