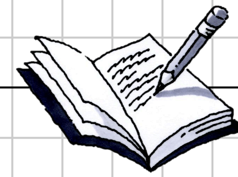
CHAPTER 4: VARIABLES AND RATIOS

Date: Lesson:	Learning Log Title:



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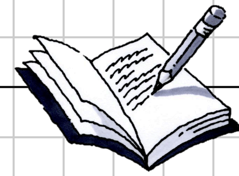

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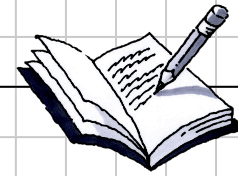


Notes:

MATH NOTES

DIVIDING

When using long division to divide one number by another, it is important to be sure that you know the place value of each digit in your result.

$$\begin{array}{r} 37 \\ 6\overline{)225} \\ -180 \\ \hline 45 \\ -42 \\ \hline 3 \end{array}$$

In the example of dividing 225 by 6 at right, people often begin by saying, *"6 goes into 22 three times."* If they were paying attention to place value, they would instead say *"6 goes into 220 thirty-something times."* The 3 of the quotient is written in the tens place to indicate that 6 goes into 225 at least 30 times, but less than 40. The 3 represents 3 tens.

$$\begin{array}{r} 37.5 \\ 6\overline{)225.0} \\ -180 \\ \hline 45 \\ -42 \\ \hline 30 \\ 30 \\ \hline 0 \end{array}$$

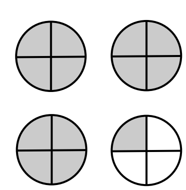
It may seem like the divisor is then multiplied by the 3, and the product, 18, is placed below a 22. However, you are really multiplying 30 by 6 and the product is 180, which is placed below 225. You would then subtract, getting what looks like 4. But then you would "bring down" the 5, to get 45. Notice that if you subtract 180 from 225, as in the top example at right, you get 45 directly. You then repeat the same process. In the past, you may have stopped at this point and written that the quotient is 37 with a remainder of 3.

The same method works for dividing decimals. The bottom example at right is essentially the same as the top one, except that it shows what happens if you keep dividing past the decimal point, while still keeping place value in mind.

MIXED NUMBERS AND FRACTIONS GREATER THAN ONE

The number $3\frac{1}{4}$ is called a **mixed number** because it is composed of a whole number, 3, and a fraction, $\frac{1}{4}$.

The number $\frac{13}{4}$ is called a **fraction greater than one** because the numerator, which represents the number of equal pieces, is larger than the denominator, which represents the number of pieces in one whole, so its value is greater than one. (Sometimes such fractions are called "improper fractions," but this is just a historical term. There is nothing actually wrong with the fractions.)



As you can see in the diagram at right, the fraction $\frac{13}{4}$ can be rewritten as $\frac{4}{4}+\frac{4}{4}+\frac{4}{4}+\frac{1}{4}$, which shows that it is equal in value to $3\frac{1}{4}$.

Your choice: Depending on which arithmetic operations you need to perform, you will choose whether to write your number as a mixed number or as a fraction greater than one.

Notes:

ADDING AND SUBTRACTING MIXED NUMBERS

To **add or subtract mixed numbers**, you can either add or subtract their parts, or you can change the mixed numbers into fractions greater than one.

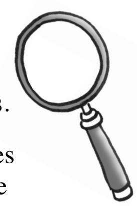
To add or subtract mixed numbers by adding or subtracting their parts, add or subtract the whole-number parts and the fraction parts separately. Adjust if the fraction in the answer would be greater than one or less than zero. For example, the sum of $3\frac{4}{5}+1\frac{2}{3}$ is calculated at right above.

$$3\tfrac{4}{5} = 3+\tfrac{4}{5}\cdot\tfrac{3}{3} = 3\tfrac{12}{15}$$
$$+1\tfrac{2}{3} = 1+\tfrac{2}{3}\cdot\tfrac{5}{5} = +1\tfrac{10}{15}$$
$$4\tfrac{22}{15} = 5\tfrac{7}{15}$$

It is also possible to add or subtract mixed numbers by first changing them into fractions greater than one. Then add or subtract in the same way you would if they were fractions between 0 and 1. For example, the sum of $2\frac{1}{6}+1\frac{4}{5}$ is calculated at right.

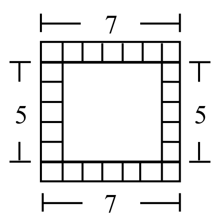
$$\begin{aligned} 2\tfrac{1}{6}+1\tfrac{4}{5} &= \tfrac{13}{6}+\tfrac{9}{5} \\ &= \tfrac{13}{6}\cdot\tfrac{5}{5}+\tfrac{9}{5}\cdot\tfrac{6}{6} \\ &= \tfrac{65}{30}+\tfrac{54}{30} \\ &= \tfrac{119}{30} \\ &= 3\tfrac{29}{30} \end{aligned}$$

USING VARIABLES TO GENERALIZE

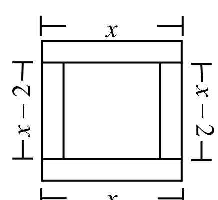


Variables are letters or symbols used to represent one or more numbers. They are often used to generalize patterns from a few specific numbers to include all possible numbers.

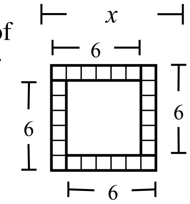
For example, if a square is surrounded by smaller square tiles each measuring one centimeter on a side, how many tiles are needed? It helps to look at a specific of size square first.

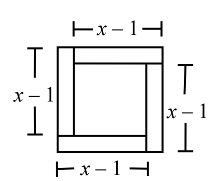


The outside square at right has side length 7. One way to see the total number of tiles needed for the frame is to consider that it needs 7 tiles for each of the top and bottom sides and 7 – 2 = 5 tiles for the left and right sides. This is shown in the first diagram at right. The total number of tiles needed for the frame can be counted as 7 + 7 + 5 + 5 = 24.



Square frames with different side lengths will follow the same pattern. You can generalize by writing an expression for any side length, denoted by x. The second diagram at right shows that the top and bottom each contain x tiles. The right and left sides each contain $x – 2$ tiles. You could write the total number of tiles as either $x + x + (x – 2) + (x – 2)$, $2x + 2(x – 2)$, or even as $4x – 4$.





Shown at right are two additional square-frame diagrams. The diagram on the left shows another way to count the number of tiles in a frame. The diagram on the right shows the algebraic expression associated with it. Notice that the expression resulting from this counting method could be written $(x – 1) + (x – 1) + (x – 1) + (x – 1)$, or $4(x –1)$.

Notes:

EVALUATING ALGEBRAIC EXPRESSIONS

An **algebraic expression**, also known as a *variable expression*, is a combination of numbers and variables, connected by mathematical operations. For example, $4x$, $3(x – 5)$, and $4x – 3y + 7$ are algebraic expressions.

Addition and subtraction separate expressions into parts called **terms**. For example the expression above, $4x – 3y + 7$, has three terms: $4x$, $–3y$, and 7.

A more complex expression is $2x + 3(5 – 2x) + 8$. It also has three terms: $2x$, $3(5 – 2x)$, and 8. But the term $3(5 – 2x)$ has another expression, $5 – 2x$, inside the parentheses. The terms of this inner expression are 5 and $–2x$.

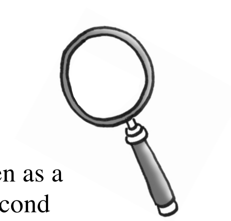
To **evaluate** an algebraic expression for particular values of variables, replace the variables in the expression with their known numerical values and simplify. Replacing variables with their known values is called **substitution**. An example is provided below.

Evaluate $4x – 3y + 7$ for $x = 2$ and $y = 1$.

Replace x and y with their known values of 2 and 1, respectively, and simplify.

$$4(2) – 3(1) + 7$$
$$8 – 3 + 7$$
$$12$$

RATIOS



A **ratio** is a comparison of two numbers, often written as a quotient; that is, the first number is divided by the second number (but not zero). A ratio can be written in words, in fraction form, or with colon notation. Most often, in this class, you will either write ratios in the form of fractions or state the ratios in words.

For example, if there are 38 students in a school band and 16 of them are boys, you can write the ratio of the number of boys to the number of girls as:

16 boys to 22 girls $\frac{16 \text{ boys}}{22 \text{ girls}}$ 16 boys : 22 girls