CHAPTER 9: ANGLES AND THE PYTHAGOREAN THEOREM

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MATH NOTES

ANGLE VOCABULARY

It is common to identify angles using three letters. For example, ∠*ABC* means the angle you would find by going from point *A* to point *B* to point *C* in the diagram at right. Point *B* is the **vertex** of the angle (where the endpoints of the two sides meet), and $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are the rays that define it. A **ray** is a part of a line that has an endpoint (starting point) and extends infinitely in one direction.
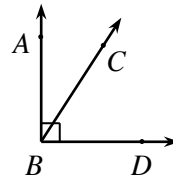
If two angles have measures that add up to 90°, they are called **complementary angles**. For example, in the diagram above, ∠*ABC* and ∠*CBD* are complementary because together they form a right angle.
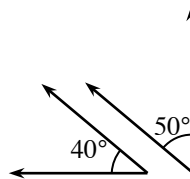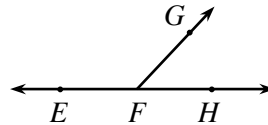
If two angles have measures that add up to 180°, they are called **supplementary angles**. For example, in the diagram below right, ∠*EFG* and ∠*GFH* are supplementary because together they form a **straight angle** (that is, together they form a line).
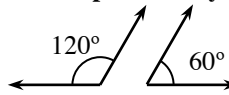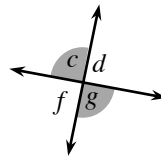
Two angles do not have to share a vertex to be complementary or supplementary.

Adjacent angles are angles that have a common vertex, share a common side, and have no interior points in common. So angles ∠*c* and ∠*d* in the diagram at right are adjacent angles, as are ∠*c* and ∠*f*, ∠*f* and ∠*g*, and ∠*g* and ∠*d*.

Vertical angles are the two opposite (that is, non-adjacent) angles formed by two intersecting lines, such as angles ∠*c* and ∠*g* in the diagram above right. By itself, ∠*c* is not a vertical angle, nor is ∠*g*, although ∠*c* and ∠*g* together are a pair of vertical angles. Vertical angles always have equal measure.

Complementary

40°   50°

Supplementary

120°   60°

A   *C*

B   *D*

*G*

E   *F*   *H*

c *d*
f *g*

Notes:

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PARALLEL LINES AND ANGLE PAIRS

Corresponding angles lie in the same position but at different points of intersection of the transversal. For example, in the diagram at right, $\angle m$ and $\angle d$ form a pair of corresponding angles, since both of them are to the right of the transversal and above the intersecting line. Corresponding angles are congruent when the lines intersected by the transversal are parallel.

$\angle f$ and $\angle m$ are **alternate interior angles** because one is to the left of the transversal, one is to the right, and both are between (inside) the pair of lines. Alternate interior angles are congruent when the lines intersected by the transversal are parallel.

$\angle g$ and $\angle m$ are **same side interior angles** because both are on the same side of the transversal and both are between the pair of lines. Same side interior angles are supplementary when the lines intersected by the transversal are parallel.
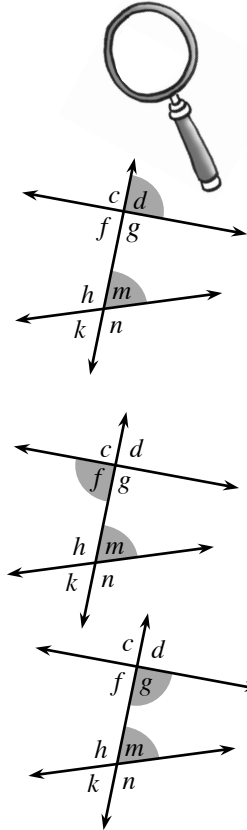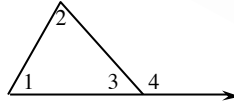
ANGLE SUM THEOREM FOR TRIANGLES

The measures of the angles in a triangle add up to 180°. For example, in ΔABC at right, $m\angle A + m\angle B + m\angle C = 180°$.

You can verify this statement by carefully drawing a triangle with a ruler, tearing off two of the angles ($\angle A$ and $\angle B$), and placing them side by side with the third angle ($\angle C$) on a straight line. The sum of the three angles is the same as the straight angle (line), that is, 180°.
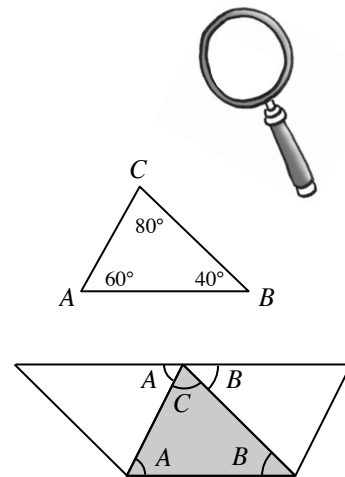
EXTERIOR ANGLE THEOREM FOR TRIANGLES

An **exterior angle** of a triangle is an angle outside of the triangle created by extending one of the sides of the triangle. In the diagram at right, ∠4 is an exterior angle.

The **Exterior Angle Theorem for Triangles** states that the measure of the exterior angle of a triangle is equal to the sum of the **remote interior angles**. In the diagram, ∠1 and ∠2 are the remote interior angles to ∠4. Note that some texts call these angles "opposite interior angles." In symbols:
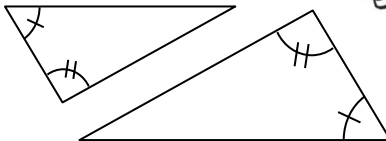
$$m\angle 4 = m\angle 1 + m\angle 2$$

AA SIMILARITY FOR TRIANGLES

For two triangles to be similar, corresponding angles must have equal measure.

However, it is sufficient to know that two pairs of corresponding angles have equal measures, because then the third pair of angles must have equal measure.

This is known as the **Angle-Angle Triangle Similarity Conjecture**, which can be abbreviated as "AA Similarity" or "AA ~."

AA ~ : If two pairs of corresponding angles have equal measure, then the triangles are similar.

Notes:

Notes:

TRIANGLE INEQUALITY AND SIDE-LENGTH PATTERNS

The **Triangle Inequality** establishes the required relationships for three lengths to form a triangle. You can also use these lengths to determine the type of triangle they form — acute, obtuse, or right — by comparing the squares of the lengths of the sides as described below.

For any three lengths to form a triangle, the sum of the lengths of any two sides must be greater than the length of the third side.

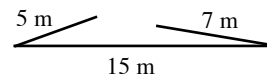For example, the lengths 3 cm, 10 cm, and 11 cm will form a triangle, because:
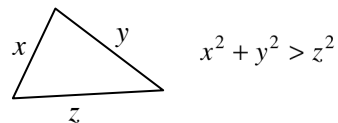
$$3 + 10 > 11$$
$$3 + 11 > 10$$
$$10 + 11 > 3$$

The lengths 5 m, 7 m, and 15 m will not form a triangle, because $5 + 7 = 12$, and $12 \not> 15$.

Acute triangle: The sum of the squares of the lengths of the two shorter sides is greater than the square of the length of the longest side.

$$x^2 + y^2 > z^2$$

Obtuse triangle: The sum of the squares of the lengths of the two shorter sides is less than the square of the length of the longest side.

$$p^2 + q^2 < r^2$$
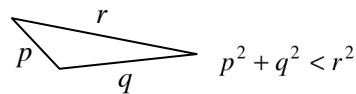
Right triangle: The sum of the squares of the lengths of the two shorter sides is equal to the square of the length of the longest side.

$$a^2 + b^2 = c^2$$
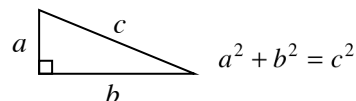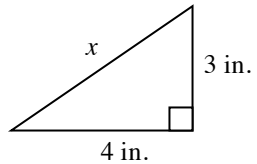
RIGHT TRIANGLES AND THE PYTHAGOREAN THEOREM

A right triangle is a triangle in which the two shorter sides form a right (90°) angle. The shorter sides are called **legs**. The third and longest side, called the **hypotenuse**, is opposite the right angle.

The **Pythagorean Theorem** states that for any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Leg 1
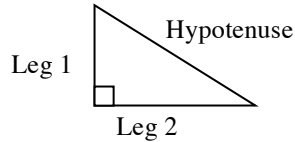
Hypotenuse

Leg 2

$$(\text{leg 1})^2 + (\text{leg 2})^2 = (\text{hypotenuse})^2$$

Example:

x

3 in.

4 in.

$$3^2 + 4^2 = x^2$$

$$9 + 16 = x^2$$

$$25 = x^2$$

$$5 = x$$

The **converse of the Pythagorean Theorem** states that if the sum of the squares of the lengths of the two shorter sides of a triangle equals the square of the length of the longest side, then the triangle is a right triangle. For example:

Do the lengths 6, 9, and 11 form a right triangle?

$$6^2 + 9^2 \overset{?}{=} 11^2$$

$$36 + 81 \overset{?}{=} 121$$

$$117 \neq 121$$

No, these lengths do not form a right triangle.

Do the lengths 9, 40, and 41 form a right triangle?

$$9^2 + 40^2 \overset{?}{=} 41^2$$

$$81 + 1600 \overset{?}{=} 1681$$

$$1681 = 1681$$

Yes, these lengths form a right triangle.

Notes:

THE REAL-NUMBER SYSTEM

The **real numbers** include all of the **rational numbers** and **irrational numbers**.

Rational numbers are numbers that can be written as a fraction in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Rational numbers written in decimal form either terminate or repeat. The number 7 is a rational number, because it can be written as $\frac{7}{1}$. The number –0.687 is rational, because it can be written as $-\frac{687}{1000}$. Even $\sqrt{25}$ is rational, because it can be written as $\frac{5}{1}$. Other examples of rational numbers include –12, 0, 3, $\frac{1}{8}$, $\frac{5}{9}$, 0.25, and $\sqrt{81}$.

Irrational numbers are numbers that cannot be written as fractions. Decimals that do not terminate or repeat are irrational numbers. For example, $\sqrt{3}$ is an irrational number. It cannot be written as a fraction, and when it is written as a decimal, it neither terminates nor repeats ($\sqrt{3} \approx 1.73205080756...$). Other irrational numbers include $\sqrt{2}$, $\sqrt{7}$, and π.
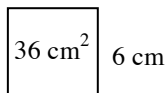
SQUARING AND SQUARE ROOTS

When a number or variable is multiplied by itself, it is said to be **squared**. Squaring a number is like finding the area of the square with that number or variable as its side length. For example:
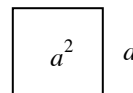
$$6 \cdot 6 = 6^2 = 36 \qquad\qquad a \cdot a = a^2$$

and

and

| 36 cm² | 6 cm |

6 cm

| a^2 | a |

a

The **square root** of a number or variable is the positive factor that, when multiplied by itself, results in the given number. Use a **radical sign**, $\sqrt{}$, to show this operation. If you know the area of a square, then the square root of the numerical value of the area is the side length of that square.

For example, $\sqrt{49}$ is read as, "the square root of 49," and means, "Find the positive number that multiplied by itself equals 49." $\sqrt{49} = 7$, since $7 \cdot 7 = 49$.

By definition, –7 is not the square root of 49 even though $(-7) \cdot (-7) = 49$, since only positive numbers are considered to be square roots. No real square region could have a negative side length.