CHAPTER 5: SYSTEMS OF EQUATIONS

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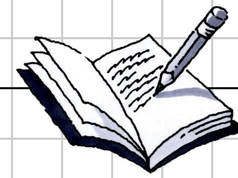

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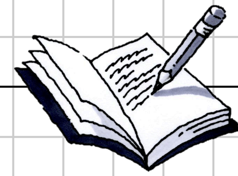

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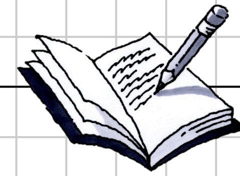


MATH NOTES

Notes:

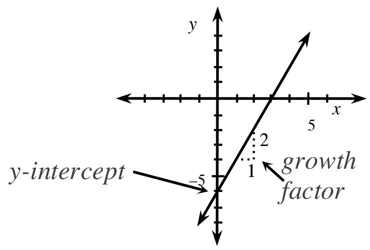
LINEAR EQUATIONS

A **linear equation** is an equation that forms a line when it is graphed. This type of equation may be written in several different forms. Although these forms look different, they are equivalent; that is, they all graph the same line.

Standard form: An equation in $ax + by = c$ form, such as $-6x + 3y = 18$.

$\boldsymbol{y = mx + b}$ **form:** An equation in $y = mx + b$ form, such as $y = 2x + 6$.

You can quickly find the **growth factor** and ***y*-intercept** of a line in $y = mx + b$ form. For the equation $y = 2x - 6$, the growth factor is 2 , while the *y*-intercept is (0, –6).



$$y = \underbrace{2}_{\textit{growth factor}}x \underbrace{- 6}_{\textit{y-intercept}}$$

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EQUIVALENT EQUATIONS

Two equations are **equivalent** if they have all the same solutions. There are many ways to change one equation into a different, equivalent equation. Common ways include: *adding* the same number to both sides, *subtracting* the same number from both sides, *multiplying* both sides by the same number, *dividing* both sides by the same (non-zero) number, and *rewriting* one or both sides of the equation.

For example, the equations below are all equivalent to $2x + 1 = 3$:

$20x + 10 = 30$	$2(x + 0.5) = 3$
$\frac{2x}{3} + \frac{1}{3} = 1$	$0.002x + 0.001 = 0.003$

Notes:

SOLVING EQUATIONS WITH FRACTIONS (ALSO KNOWN AS THE FRACTION BUSTER METHOD)

Example: Solve $\frac{x}{3}+\frac{x}{5}=2$ for x.

This equation would be much easier to solve if it had no fractions. Therefore, the first goal is to find an equivalent equation that has no fractions.

To eliminate the denominators, multiply both sides of the equation by the common denominator. In this example, the lowest common denominator is 15, so multiplying both sides of the equation by 15 eliminates the fractions. Another approach is to multiply both sides of the equation by one denominator and then by the other.

$$\frac{x}{3}+\frac{x}{5}=2$$

The lowest common denominator of $\frac{x}{3}$ and $\frac{x}{5}$ is 15.

$$15\cdot\left(\frac{x}{3}+\frac{x}{5}\right)=15\cdot 2$$

$$15\cdot\frac{x}{3}+15\cdot\frac{x}{5}=15\cdot 2$$

Either way, the result is an equivalent equation without fractions:

$$5x+3x=30$$

$$8x=30$$

$$x=\frac{30}{8}=\frac{15}{4}=3.75$$

The number used to eliminate the denominators is called a **Fraction Buster**. Now the equation looks like many you have seen before, and it can be solved in the usual way.

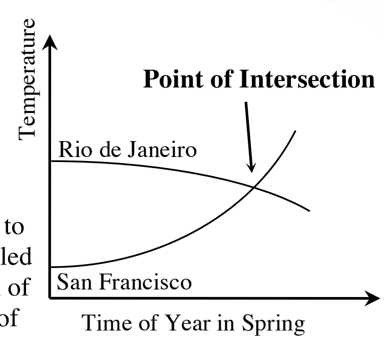
Once you have found the solution, remember to check your answer.

$$\frac{3.75}{3}+\frac{3.75}{5}=2$$

$$1.25+0.75=2$$

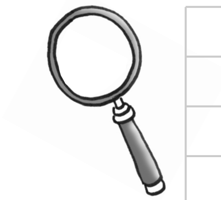
SYSTEM OF EQUATIONS VOCABULARY

The point where two lines or curves intersect is called a **point of intersection**. This point's significance depends on the context of the problem.



Two or more lines or curves used to find a point of intersection are called a **system of equations**. A system of equations can represent a variety of contexts and can be used to compare how two or more things are related. For example, the system of equations graphed above compares the temperature in two different cities over time.

THE EQUAL VALUES METHOD



The **Equal Values Method** is a non-graphing method to find the point of intersection or solution to a system of equations.

Start with two equations in $y = mx + b$ form, such as $y = -2x + 5$ and $y = x - 1$. Take the two expressions that equal y and set them equal to each other. Then solve this new equation to find x. See the example at right.

$$-2x + 5 = x - 1$$
$$-3x = -6$$
$$x = 2$$

Once you know the x-coordinate of the point of intersection, substitute your solution for x into *either* original equation to find y. In this example, the first equation is used.

$$y = -2x + 5$$
$$y = -2(2) + 5$$
$$y = 1$$

A good way to check your solution is to substitute your solution for x into *both* equations to verify that you get equal y-values.

$$y = x - 1$$
$$y = (2) - 1$$
$$y = 1$$

Write the solution as an ordered pair to represent the point on the graph where the equations intersect.

$$(2, 1)$$

SOLUTIONS TO A SYSTEM OF EQUATIONS

A **solution** to a system of equations gives a value for each variable that makes both equations true. For example, when 4 is substituted for x and 5 is substituted for y in both equations at right, both equations are true. So $x = 4$ and $y = 5$ or (4, 5) is a solution to this system of equations. When the two equations are graphed, (4, 5) is the point of intersection.

System with one solution: intersecting lines

$$x - y = -1$$
$$2x - y = 3$$

Some systems of equations have no solutions or infinite solutions. Consider the examples at right.

System with no solution: parallel lines

$$x + y = 3$$
$$x + y = 4$$

Notice that the Equal Values Method would yield $3 = 4$, which is never true. When the lines are graphed, they are parallel. Therefore, the system has **no solution**.

System with infinite solutions: coinciding lines

$$x + y = 3$$
$$2x + 2y = 6$$

In the third set of equations, the second equation is just the first equation multiplied by 2. Therefore, the two lines are really the same line and have **infinite solutions**.

Notes: