# CHAPTER 6: SOLVING INEQUALITIES AND EQUATIONS

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| Date: Lesson: | Learning Log Title: |
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# MATH NOTES

## INEQUALITY SYMBOLS

Just as the symbol "=" is used in mathematics to represent that two quantities are equal, the **inequality symbols** at right are used to describe the relationships between quantities that are not necessarily equal. Examples: $3 < 7$, $14 \leq 14$, $-7 < -3$, $19 \geq 14$.

$<$  less than

$\leq$  less than or equal to

$>$  greater than

$\geq$  greater than or equal to

## ALGEBRA VOCABULARY

**Variable:** A letter or symbol that represents one or more numbers.

**Expression:** A combination of numbers, variables, and operation symbols. An expression does not contain an equal sign. For example, $2x + 3(5 - 2x) + 8$. Also, $5 - 2x$ is a smaller expression within the larger expression.

**Term:** Parts of the expression separated by addition and subtraction. For example, in the expression $2x + 3(5 - 2x) + 8$, the three terms are $2x$, $3(5 - 2x)$, and 8. The expression $5 - 2x$ has two terms, 5 and $2x$.

**Coefficient:** The numerical part of a term. In the expression $2x + 3(5 - 2x) + 8$, 2 is the coefficient of $2x$. In the expression $17x - 15x^2$, both 7 and 15 are coefficients.

**Constant term:** A number that is not multiplied by a variable. In the example above, 8 is a constant term. The number 3 is not a constant term because it is multiplied by a variable inside the parentheses.

**Factor:** Part of a multiplication expression. In the expression $3(5 - 2x)$, 3 and $5 - 2x$ are factors.
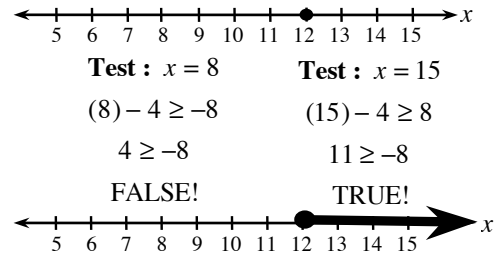
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# GRAPHING INEQUALITIES

To solve and graph an inequality with one variable, first treat the problem as if it were an equality and solve the problem. The solution to the equality is called the **boundary point**. For example, to solve $x - 4 \geq 8$, first solve $x - 4 = 8$. The solution $x = 12$ is the boundary point for the inequality $x - 4 \geq 8$.

Since the original inequality is true when $x = 12$, place your boundary point on the number line as a solid point. Then test one value on either side in the *original* inequality by substituting it into the original inequality.

$$5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15$$

**Test :** $x = 8$      **Test :** $x = 15$

$(8) - 4 \geq -8$      $(15) - 4 \geq 8$

$4 \geq -8$      $11 \geq -8$

FALSE!      TRUE!

$$5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15$$

This will determine which set of numbers makes the inequality true. Write the inequality solution and extend an arrow onto the number line in the direction of the side that makes the inequality true. This is shown with the examples of $x = 8$ and $x = 15$ above. Therefore, the solution is $x \geq 12$ (also shown on the number line).

When the inequality is $<$ or $>$, the boundary point is *not* included in the answer. On a number line, this would be indicated with an open circle at the boundary point. For example, the graph of $x < 7$ is shown below.
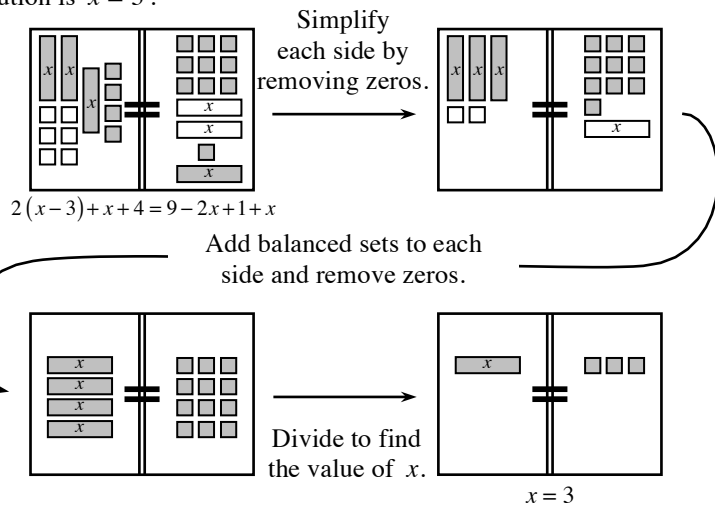
$$-1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7$$
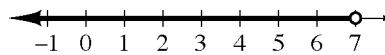
# USING AN EQUATION MAT

An **Equation Mat** can help you visually represent an equation with algebra tiles. It can also help you find the solution to an equation.

For example, the equation $2(x - 3) + x + 4 = 9 - 2x + 1 + x$ can be represented as shown on the first Equation Mat below. Then it can be solved using simplification steps (also know as legal moves) to show that the solution is $x = 3$.

Simplify each side by removing zeros.

$2(x - 3) + x + 4 = 9 - 2x + 1 + x$

Add balanced sets to each side and remove zeros.

Divide to find the value of $x$.
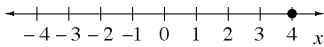
$x = 3$

# EQUATIONS AND INEQUALITIES

**Equations** always have an equal sign. **Inequalities** have one of inequality symbols defined in the Lesson 6.1.1 Math Note. To **solve** an equation or inequality means to find all values of the variable that make the relationship true. The solution can be shown on a number line. See the examples below.

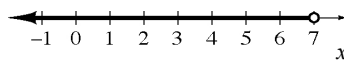**Solve** this equation:

$$x + 3 = 7$$

The solution is:

$$x = 4$$

**Solve** this inequality:

$$x - 2 < 5$$

The solution is:

$$x < 7$$

# CHECKING A SOLUTION

To check a solution to an equation, substitute the solution into the equation and verify that it makes the two sides of the equation equal.

For example, to verify that $x = 10$ is a solution to the equation $3(x - 5) = 15$, substitute 10 into the equation for $x$ and then verify that the two sides of the equation are equal.

$$3(10 - 5) \overset{?}{=} 15$$
$$3(5) \overset{?}{=} 15$$
$$15 = 15$$

✔ *True, so x = 10 is a solution.*

$$3(2 - 5) \overset{?}{=} 15$$
$$3(-3) \overset{?}{=} 15$$
$$-9 \neq 15$$

✘ *Not true, so x = 2 is not a solution.*

As shown at right, $x = 10$ is a solution to the equation $3(x - 5) = 15$.

What happens when you do this check if your answer is incorrect? For example, try substituting $x = 2$ into the same equation. The result shows that $x = 2$ is not a solution to this equation.

# DEFINING A VARIABLE

When you write an equation, it is important to **define the variable** carefully. You need to be clear about what you are talking about so that someone else looking at your work understands what the variable represents. This step is an important habit to develop because it is an important step in solving many different math problems.

For example, suppose you have this problem:

At the neighborhood grocery store, grapes cost $3 a pound. If Belinda spent $5.40 on grapes, how many pounds of grapes did she buy?

One equation you could write is $3x = 5.4$, if you know what $x$ stands for. The variable $x$ should be clearly defined, such as $x =$ pounds of grapes, rather than just $x =$ grapes. You could also write $g =$ pounds of grapes, since any letter may be used as a variable.

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# SOLUTIONS TO AN EQUATION WITH ONE VARIABLE

A **solution** to an equation gives the value(s) of the variable that makes the equation true.

For example, when 5 is substituted for $x$ in the equation at right, both sides of the equation are equal. Therefore, $x = 5$ is a solution to this equation. Some equations have several solutions, such as $x^2 = 25$, where $x = 5$ or $-5$.

$$4x - 1 = 2x + 9$$
$$4(5) - 1 = 2(5) + 9$$
$$19 = 19$$

Equations may also have no solution or an infinite (unlimited) number of solutions.

Notice that no matter what the value of $x$ is, the left side of the first equation will never equal the right side. Therefore, it could be said that $x + 2 = x + 3$ has **no solution**.

Equation with no solution:
$$x + 2 = x + 3$$
$$2 \neq 3$$

However, in the equation $x - 2 = x - 2$, no matter what value $x$ has, the equation will always be true. All numbers can make $x - 2 = x - 2$ true. Therefore, it could be said that the solution for the equation $x - 2 = x - 2$ is **all numbers**.

Equation with infinitely many solutions:
$$x - 2 = x - 2$$
$$-2 = -2$$