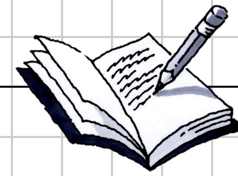
CHAPTER 5: PROBABILITY AND SOLVING WORD PROBLEMS

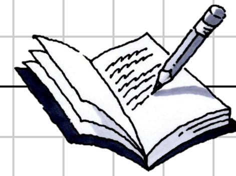
Date:
Lesson:

Learning Log Title:



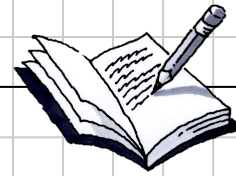
Date:
Lesson:

Learning Log Title:



Date:
Lesson:

Learning Log Title:



MATH NOTES

EQUIVALENT RATIOS

A **ratio** is a comparison of two quantities by division. A ratio can be written in words, as a fraction, or with colon notation. Most often in this course, ratios will be written as fractions or stated in words.

For example, if there are 28 students in a math class and 15 of them are girls, you can write the ratio of the number of girls to the number of students in the class as:

15 girls to 28 students $\qquad \frac{15 \text{ girls}}{28 \text{ students}} \qquad$ 15 girls : 28 students

You used a Giant One to write equivalent fractions in Chapter 1. To rewrite any ratio as an **equivalent ratio**, write it as a fraction and multiply it by a fraction equal to one. For example, you can show that the ratio of raisins to peanuts is the same for a larger mixture using a Giant One like this:

$$\frac{4 \text{ raisins}}{7 \text{ peanuts}} \cdot \frac{20}{20} = \frac{80 \text{ raisins}}{140 \text{ peanuts}}$$

Equivalent fractions (or ratios) can be thought of as families of fractions. There are an infinite number of fractions that are equivalent to a given fraction. You may want to review the basis for using a Giant One — the Multiplicative Identity — in the Math Notes box in Lesson 1.2.5.

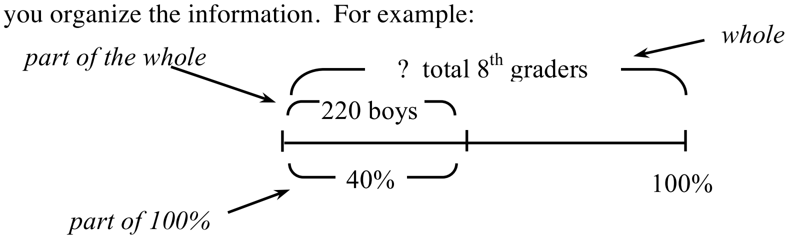
Notes:

Notes:

PART-TO-WHOLE RELATIONSHIPS

Percentages, fractions, and decimals are all different ways to represent a portion of a whole or a number. Portion-whole relationships can also be described in words.

You can represent a part-to-whole relationship with a linear model like the one below. To solve a percentage problem described in words, you must first identify three important quantities: the percent, the whole, and the part of the whole. One of the quantities will be unknown. A diagram can help you organize the information. For example:



Once the parts have been identified, you can use reasoning to extend the part to the whole. For example, if 220 students are 40% of eighth graders, then 10% must be $220 \div 4 = 55$. Then 100% must be $55 \cdot 10 = 550$ students. Another way to solve the problem is to find the ratio of 220 boys to the whole (all students) and compare that ratio to 40% and 100%. This could be written:

$$\frac{40}{100} \cdot \frac{}{} = \frac{220}{?} \text{ , then } \frac{40}{100} \cdot \frac{5.5}{5.5} = \frac{220}{?}$$

You can see above that the total number of 8^{th} graders is 550.

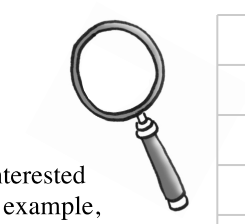
To remember how to rewrite decimals or fractions as percents, and to rewrite percents as fractions or decimals, refer to the Math Notes box at the end of Lesson 1.3.1.

INDEPENDENT AND DEPENDENT EVENTS

Two events are **independent** if the outcome of one event does not affect the outcome of the other event. For example, if you draw a card from a standard deck of playing cards but replace it before you draw again, the outcomes of the two draws are independent.

Two events are **dependent** if the outcome of one event affects the outcome of the other event. For example, if you draw a card from a standard deck of playing cards and do not replace it for the next draw, the outcomes of the two draws are dependent.

PROBABILITY OF COMPOUND EVENTS



Sometimes when you are finding a probability, you are interested in either of two outcomes taking place, but not both. For example, you may be interested in drawing a king or a queen from a deck of cards. At other times, you might be interested in one event followed by another event. For example, you might want to roll a one on a number cube and then roll a six. The probabilities of combinations of simple events are called **compound events**.

To find the probability of *either* one event *or* another event that has nothing in common with the first, you can find the probability of each event separately and then add their probabilities. Using the example above of drawing a king or a queen from a deck of cards:

$$\text{P(king)} = \tfrac{4}{52} \quad \text{and} \quad \text{P(queen)} = \tfrac{4}{52} \quad \text{so} \quad \text{P(king or queen)} = \tfrac{4}{52} + \tfrac{4}{52} = \tfrac{8}{52} = \tfrac{2}{13}$$

For two independent events, to find the probability of *both* one *and* the other event occurring, you can find the probability of each event separately and then multiply their probabilities. Using the example of rolling a one followed by a six on a number cube:

$$\text{P(1)} = \tfrac{1}{6} \quad \text{and} \quad \text{P(6)} = \tfrac{1}{6} \quad \text{so} \quad \text{P(1 then 6)} = \tfrac{1}{6} \cdot \tfrac{1}{6} = \tfrac{1}{36}$$

Note that you would carry out the same computation if you wanted to know the probability of rolling a one on a green cube and a six on a red cube if you rolled both of them at the same time.

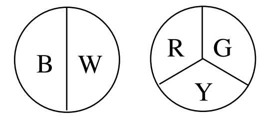
Notes:

Notes:

PROBABILITY MODELS FOR MULTIPLE EVENTS

To determine all possible outcomes for multiple events when *both* one event *and* the other occur, there are several different models you can use to help organize the information.

Consider spinning each spinner at right once.



If you use a plan or a pattern to find all of the outcomes in an event, you are making a **systematic list**. For example, assume that you first spin B on spinner 1. Then, list all of the possible outcomes on spinner 2. Next, assume that your first spin is W on spinner 1, and complete the list.

Systematic List

BR	WR
BG	WG
BY	WY

A **probability table** can also organize information if there are exactly two events. The possibilities for each event are listed on the sides of the table as shown, and the combinations of outcomes are listed inside the table. In the example at right, the possible outcomes for spinner 1 are listed on the left side, and the possible outcomes for spinner 2 are listed across the top.

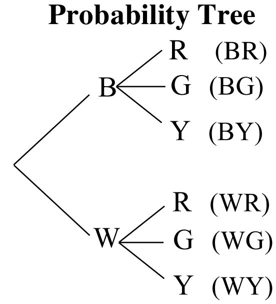
Probability Table

	R	G	Y
B	BR	BG	BY
W	WR	WG	WY

The possible outcomes of the two events are shown inside the rectangle. In this table, the top and side are divided evenly because the outcomes are equally likely. Inside the table you can see the possible combinations of outcomes.

A **probability tree** is another method for organizing information. The different outcomes are organized at the end of branches of a tree. The first section has B and W at the ends of two branches because there are two possible outcomes of spinner 1, namely B and W. Then the ends of three more branches represent the possible outcomes of the second spinner, R, G, and Y. These overall possible outcomes of the two events are shown as the six branch ends.

Probability Tree

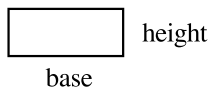


SOLVING PROBLEMS WITH THE 5-D PROCESS

The **5-D Process** is an organized method to solve problems. The D's stand for Describe/Draw, Define, Do, Decide, and Declare. An example of this work is shown below.

Problem: The base of a rectangle is 13 centimeters longer than the height. If the perimeter is 58 centimeters, find the base and the height of the rectangle.

Describe/Draw: The shape is a rectangle and we are looking at the perimeter.



	Define		Do	Decide
	Height (trial)	Base (height + 13)	Perimeter 2(base) + 2(height)	58?
Trial 1:	10	10 + 13 = 23	2(23) + 2(10) = 66	66 is too high

Use a trial value.

Use the relationships stated in the problem to determine the values of the other quantities (such as base and perimeter).

Decide if the answer is correct. Revise and make another trial until you find the correct answer.

Trial 3:	7	7 + 13 = 20	2(20) + 2(7) = 54	too low
Trial 2:	8	8 + 13 = 21	2(21) + 2(8) = 58	correct

Declare: The base is 21 centimeters and the height is 8 centimeters.

CONSECUTIVE INTEGERS

Consecutive integers are integers that come "one after another" in order (that is, without skipping any of them). For example: 11, 12, and 13 are three consecutive integers. The numbers 10, 12, 14, and 16 are four **consecutive even integers** because in counting up from 10, no even numbers are skipped. Likewise, 15, 17, and 19 are **consecutive odd integers**.

In algebra, it is sometimes necessary to represent a list of consecutive integers. To represent any list in general, you must use variables. It is common to let x represent the first integer. See the examples below of how to write a list of consecutive integers.

Three consecutive integers: $x, x+1, x+2$
Three consecutive odd integers: $x, x+2, x+4$
Three consecutive even integers: $x, x+2, x+4$

Note that consecutive even integers and odd integers look alike because both even integers and odd integers are two apart.

Notes: