CHAPTER 3: ARITHMETIC PROPERTIES

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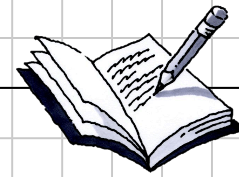

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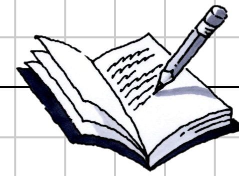
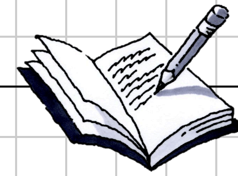


Date: Lesson:	Learning Log Title: 

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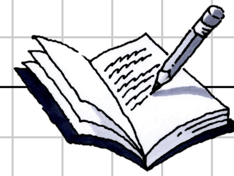

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MATH NOTES

EXPRESSIONS, TERMS, AND ORDER OF OPERATIONS

A mathematical **expression** is a combination of numbers, variables, and operation symbols. Addition and subtraction separate expressions into parts called **terms.** For example, $4x^2 - 3x + 6$ is an expression. It has three terms: $4x^2$, $3x$, and 6.

A more complex expression is $2x + 3(5 - 2x) + 8$, which also has three terms: $2x$, $3(5 - 2x)$, and 8. But the term $3(5 - 2x)$ has another expression, $5 - 2x$, inside the parentheses. The terms of this expression are 5 and $2x$.

Mathematicians have agreed on an **order of operations** simplifying expressions.

Original expression:	$(10 - 3 \cdot 2) \cdot 2^2 - \frac{13-3^2}{2} + 6$
Circle expressions that are grouped within parentheses or by a fraction bar:	$(10 - 3 \cdot 2) \cdot 2^2 - \frac{13-3^2}{2} + 6$
Simplify *within* circled terms using the order of operations:	$(10 - 3 \cdot 2) \cdot 2^2 - \frac{13-3 \cdot 3}{2} + 6$
Evaluate exponents.	$(10 - 6) \cdot 2^2 - \frac{13-9}{2} + 6$
Multiply and divide from left to right.	$(4) \cdot 2^2 - \frac{4}{2} + 6$
Combine terms by adding and subtracting from left to right.	$4 \cdot 2^2 - \frac{4}{2} + 6$
Circle the remaining terms:	$4 \cdot 2 \cdot 2 - \frac{4}{2} + 6$
Simplify *within* circled terms using the order of operations as described above:	$16 - 2 + 6$
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EVALUATING ALGEBRAIC EXPRESSIONS

An **algebraic expression** consists of one or more variables, or a combination of numbers and variables connected by mathematical operations. Examples of algebraic expressions include $4x$, $3(x-5)$, and $4x-3y+7$.

To **evaluate** an algebraic expression for particular values of the variables, replace the variables in the expression with their known numerical values and simplify. Replacing variables with their known values is called **substitution**. An example is provided below.

Evaluate $4x-3y+7$ for $x=2$ and $y=1$.

Replace x and y with their known values of 2 and 1, respectively, and simplify.

$$
\begin{aligned}
&4(2)-3(1)+7\\
&=8-3+7\\
&=12
\end{aligned}
$$

SUBTRACTING INTEGERS

One method of adding integers, mentioned in a previous Math Notes box, was to start with a diagram of the first integer, add the second integer to the diagram, eliminate zeros, and then record what is left. One method of subtracting integers is to do the same, except that instead of adding the second integer, you remove the second integer. Sometimes this removal will require adding extra zeros to the diagram. Look at the examples below:

Example 1: $-3-(-2)$

$- \quad - \quad -$

Remove 2 negatives

$- \quad - \ -$

$-3-(-2)=-1$

Example 2: $-5-(2)$

$- \quad - \quad - \quad - \quad -$

Cannot remove 2 positives

$- \quad - \quad - \quad - \quad - \quad + \quad + \quad - \quad -$

Add zeros until you can remove 2 positives

$- \quad - \quad - \quad - \quad - \quad + \ + \quad - \quad -$

$-5-(2)=-7$

Example 3: $3-(-3)$

$+ \quad + \quad +$

Cannot remove 3 negatives

$+ \quad + \quad + \quad + \ + \ + \quad - \ - \ -$

Add zeros until you can remove 3 negatives

$+ \quad + \quad + \quad + \ + \ + \quad - \ - \ -$

$3-(-3)=6$

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CONNECTING ADDITION AND SUBTRACTION OF INTEGERS

Another method for subtracting integers is to notice the relationship between addition problems and subtraction problems, as shown below:

$$-3-(-2)=-1 \quad \text{and} \quad -3+2=-1$$
$$-5-(2)=-7 \quad \text{and} \quad -5+(-2)=-7$$
$$3-(-3)=6 \quad \text{and} \quad 3+3=6$$
$$2-(-8)=10 \quad \text{and} \quad 2+8=10$$

These relationships happen because removing a negative amount gives an identical result to adding the same positive amount and vice versa. The result of subtraction of two integers is the same as the result of the addition of the first integer and the *opposite* (more formally, the **additive inverse**) of the second integer.

Example 1: $-2-(7)=-2+(-7)=-9$

Example 2: $2-(-3)=2+(3)=5$

Example 3: $-8-(-5)=-8+(5)=-3$

Example 4: $2-(9)=2+(-9)=-7$

Multiplication of Integers

Multiplication by a positive integer can be represented by combining groups of the same number:

$$(4)(3)=3+3+3+3=12 \quad \text{and} \quad (4)(-3)=-3+(-3)+(-3)+(-3)=-12$$

In both examples, the 4 indicates the number of groups of 3 (first example) and –3 (second example) to combine.

Multiplication by a negative integer can be represented by removing groups of the same number:

$$(-4)(3)=-(3)-(3)-(3)-(3)=-12$$
means "remove four groups of 3."

$$(-4)(-3)=-(-3)-(-3)-(-3)-(-3)=12$$
means "remove four groups of –3."

In all cases, if there are an *even* number of negative factors to be multiplied, the product is *positive*; if there are an *odd* number of negative factors to be multiplied, the product is *negative*.

This rule also applies when there are more than two factors. Multiply the first pair of factors, then multiply that result by the next factor, and so on, until all factors have been multiplied.

$$(-2)(3)(-3)(-5)=-90 \quad \text{and} \quad (-1)(-1)(-2)(-6)=12$$

Multiplicative Inverses and Reciprocals

Two numbers with a product of 1 are called **multiplicative inverses**.

$\frac{8}{5} \cdot \frac{5}{8} = \frac{40}{40} = 1 \qquad 3\frac{1}{4} = \frac{13}{4}$, so $3\frac{1}{4} \cdot \frac{4}{13} = \frac{13}{4} \cdot \frac{4}{13} = \frac{52}{52} = 1 \qquad \frac{1}{7} \cdot 7 = 1$

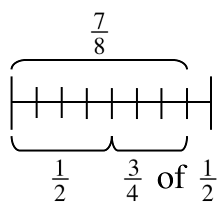
In general $a \cdot \frac{1}{a} = 1$ and $\frac{a}{b} \cdot \frac{b}{a} = 1$, where neither a nor b equals zero. You can say that $\frac{1}{a}$ is the **reciprocal** of a and $\frac{b}{a}$ is the reciprocal of $\frac{a}{b}$. Note that 0 has no reciprocal.

Division with Fractions

Method 1: Using diagrams.

To divide by a fraction using a diagram, create a model of the situation using rectangles, a linear model, or a visual representation of it. Then break that model into the fractional parts named.

For example, to divide $\frac{7}{8} \div \frac{1}{2}$, you can draw the diagram at right to visualize how many $\frac{1}{2}$-sized pieces fit into $\frac{7}{8}$. The diagram shows that one $\frac{1}{2}$ fits, with $\frac{3}{8}$ of a whole left. Since $\frac{3}{8}$ is $\frac{3}{4}$ of $\frac{1}{2}$, you can see that $1\frac{3}{4}$ $\frac{1}{2}$-sized pieces fit into $\frac{7}{8}$, so $\frac{7}{8} \div \frac{1}{2} = 1\frac{3}{4}$.



Method 2: Using common denominators.

To divide a number by a fraction using common denominators, express both numbers as fractions with the same denominator. Then divide the first numerator by the second. An example is shown at right.

$$\begin{aligned}\frac{2}{5} \div \frac{3}{10} &= \frac{4}{10} \div \frac{3}{10} \\ &= \frac{4}{3} \\ &= 1\frac{1}{3}\end{aligned}$$

Method 3: Using a Super Giant One.

To divide by a fraction using a Super Giant One, write the two numbers (dividend and divisor) as a complex fraction with the dividend as the numerator and the divisor as the denominator. Use the reciprocal of the complex fraction's denominator to create a Super Giant One. Then simplify as shown in the following example.

$$\frac{3}{4} \div \frac{2}{5} = \frac{\frac{3}{4}}{\frac{2}{5}} \cdot \boxed{\frac{\frac{5}{2}}{\frac{5}{2}}} = \frac{\frac{3}{4} \cdot \frac{5}{2}}{1} = \frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8} = 1\frac{7}{8}$$

Division with fractions by the Super Giant One method can be generalized and named the **invert and multiply** method. To invert and multiply, multiply the first fraction (dividend) by the reciprocal (or multiplicative inverse) of the second fraction (divisor). If the first number is an integer, write it as a fraction with a denominator of 1. If it is a mixed number, write it as a fraction greater than one. Here is the same problem in the third example above solved using this method:

$$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8} = 1\frac{7}{8}$$

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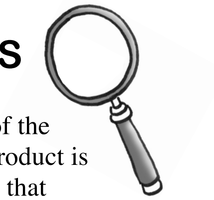
MULTIPLYING DECIMAL NUMBERS



The answer to a multiplication problem is called the product of the factors. One way to place the decimal point correctly in the product is to count the decimal places in each of the factors. Then count that many places to the left from the farthest-right digit in the product.

Examples:

one place · two places = three places   1 + 2 places = 3 places

$2.3 \cdot 5.06 = 11.638$

four places · two places = six places   4 + 2 places = 6 places

$0.0004 \cdot 3.42 = 0.001368$

DIVIDING DECIMAL NUMBERS

When you are dividing by a decimal number, one way to proceed is to count how many digits the decimal point must move to the right in the divisor so that it becomes an integer (whole number).

Then move the decimal point in the dividend the same direction and the same number of digits.

Example: $8.3 \div 4.07$

divisor $\longrightarrow$ $4.07\overline{)8.30}$ $\longleftarrow$ dividend

Moving the decimal point two places to the right is the same as multiplying both numbers by 100.

The Giant One (Identity Property of Multiplication) illustrates this as shown below.

$$8.3 \div 4.07 = \frac{8.3}{4.07} \cdot \frac{100}{100} = \frac{830}{407}$$