Chapter 2: Fractions and Integer Addition

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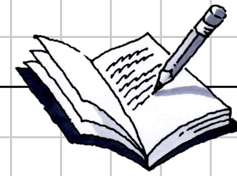

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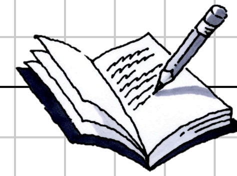

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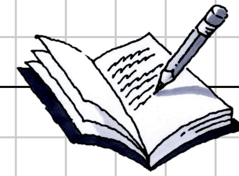


MATH NOTES

Notes:

MIXED NUMBERS AND FRACTIONS GREATER THAN ONE

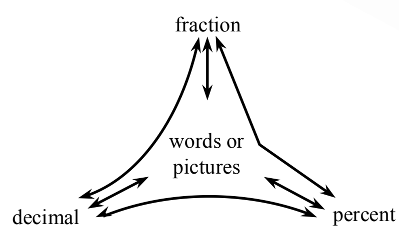
The number $4\frac{1}{3}$ is called a **mixed number** because it is composed of a whole number, 4, and a fraction, $\frac{1}{3}$.

The number $\frac{13}{3}$ is a called a **fraction greater than one** because the numerator is larger than the denominator and its value is therefore greater than one. It is equal to the mixed number $4\frac{1}{3}$. Sometimes fractions greater than one are called *improper fractions*, but this is just a historical term. There is nothing actually wrong with the fraction.

Whether to write a number as a mixed number or a fraction greater than one depends on what arithmetic operation(s) you are performing. For some arithmetic operations, especially multiplication and division, you will usually want to write mixed numbers as fractions greater than one.

Fraction⇔Decimal⇔Percent

The **Representations of a Portion web** diagram at right illustrates that fractions, decimals, and percents are different ways to represent a portion of a number. Portions can also be represented in words, such as "four fifths" or "twelve-fifteenths" or with diagrams.



Representations of a Portion

Decimal to percent:
Multiply the decimal by 100.
$(0.34)(100) = 34\%$

Fraction to percent:
Set up an equivalent fraction using 100 as the denominator. The numerator is the percent.
$\frac{4}{5} \cdot \frac{20}{20} = \frac{80}{100} = 80\%$

Terminating decimal to fraction:
Use the digits as the numerator. Use the decimal place value as the denominator. Simplify as needed.
$0.2 = \frac{2}{10} = \frac{1}{5}$

Fraction to decimal:
Divide the numerator by the denominator.
$\frac{3}{8} = 3 \div 8 = 0.375$

Percent to decimal:
Divide the percent by 100.
$78.6\% = 78.6 \div 100 = 0.786$

Percent to fraction:
Use 100 as the denominator. Use the number in the percent as the numerator. Simplify as needed.
$22\% = \frac{22}{100} \cdot \frac{1/2}{1/2} = \frac{11}{50}$

Repeating decimal to fraction:
Count the number of decimal places in the repeating block. Write the repeating block as the numerator. Then, write the power of 10 for the number of places in the block, less 1, as the denominator. Below, the repeating block (713) has 3 decimal places so 713 is the numerator and $1000 \underline{-1}$ is the denominator.
$0.713 = \frac{713}{1000-1} = \frac{713}{999}$

Notes:

MULTIPLICATION USING GENERIC RECTANGLES

To prepare for later topics in this course and future courses it is helpful to use an area model or generic rectangle to represent multiplication.

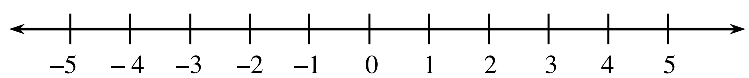
For the problem 67 • 46, think of 67 as 60 + 7 and 46 as 40 + 6. Use these numbers as the dimensions of a large rectangle as shown at right. Determine the area of each of the smaller rectangles and then find the sum of the four smaller areas. This sum is the answer to the original problem.

	60	7
40	2400	280
6	360	42

$$67 \cdot 46 = (60 + 7)(40 + 6) = 2400 + 280 + 360 + 42 = 3082$$

INTEGERS

Integers are positive and negative whole numbers and zero. On a number line, think of integers as "whole steps or no steps" in either direction from 0.



ADDITIVE INVERSE AND ADDITIVE IDENTITY

The **additive identity** is the number zero (0). When you add zero to any number, you get the same number you started with. For example, $3 + 0 = 3$. In general, $x + 0 = x$.

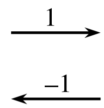
The **additive inverse** of a number is its opposite. For example, the additive inverse of 7 is –7 and the additive inverse of –2 is 2. The additive inverse "undoes" addition. Suppose you have the number 3 and you want to add a number to it to get zero (the additive identity). Then adding –3 gives you $3 + (-3) = 0$. Thus, –3 is the additive inverse of 3, and 3 is the additive inverse of –3. In general, $x + (-x) = 0$.

Notes:

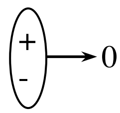
ADDITION OF INTEGERS

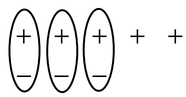
Recall that **integers** are positive and negative whole numbers and zero, that is, … –3, –2, –1, 0, 1, 2, 3, … .

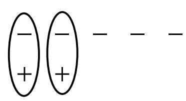
You have been introduced to two ways to think about addition. Both of them involve figuring out which, if any, parts of the numbers combine to form zero. One way to think about this concept is to think about the tightrope walker from problem 2-31. If Cecil travels one foot to the right (+1) and one foot to the left (–1), he will end up where he started, so the sum of (+1) and (–1) is zero.



Another useful strategy for finding zero is to use + and – tiles. The diagram at right can be represented by the equation $-1+1=0$. You can use this same idea for **adding any two integers**. Use + and – tiles to build the first integer, add the tiles for the second integer, and then eliminate zeros. Study the examples shown in the diagrams below.



Example 1: $5+(-3)$  $5+(-3)=2$

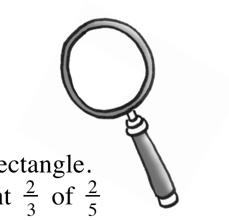
Example 2: $-5+(2)$  $-5+(2)=-3$

Example 3: $-6+(-2)$ $-6+(-2)=-8$

With practice, zeros can be visualized. This helps you determine how many remaining positive or negative tiles show the simplified expression.

Notes:

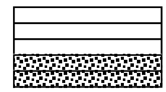
MULTIPLYING FRACTIONS USING A RECTANGLE



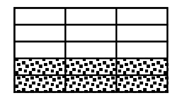
One way to model multiplying fractions is to shade a unit rectangle. Below is an example of shading a unit rectangle to represent $\frac{2}{3}$ of $\frac{2}{5}$ or, written as multiplication, $\frac{2}{3} \cdot \frac{2}{5}$.

Step 1: Divide a rectangle into five sections ("fifths")—the denominator of the second fraction. (Notice that the second number has been drawn first.)

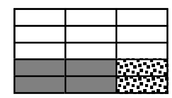
Step 2: Shade horizontal sections to represent how many fifths there are—the numerator of the second fraction.



Step 3: Divide the rectangle vertically using the denominator of the other factor ("thirds").



Step 4: Use a darker shading to show how many thirds there are. For this example, shade two-thirds of the two-fifths.



Step 5: The product's numerator is the number of sections that are double-shaded. The product's denominator is the total number of sections in the rectangle. Write an equation to show the product: $\frac{2}{3} \cdot \frac{2}{5} = \frac{4}{15}$. Simplify or reduce the product when possible.

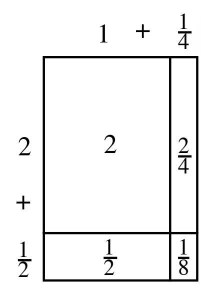
MULTIPLYING MIXED NUMBERS

Method 1: Using a generic rectangle.

To multiply mixed numbers, you may use a generic rectangle (based on the Distributive Property).

Example: $2\frac{1}{2} \cdot 1\frac{1}{4}$

$$(2+\tfrac{1}{2}) \cdot (1+\tfrac{1}{4}) = 2 + \tfrac{2}{4} + \tfrac{1}{2} + \tfrac{1}{8}$$
$$= 2 + \tfrac{1}{2} + \tfrac{1}{2} + \tfrac{1}{8}$$
$$= 2 + 1 + \tfrac{1}{8}$$
$$= 3\tfrac{1}{8}$$



Find the area of each small rectangle.

Method 2: Using fractions greater than one. To multiply mixed numbers, first change them to fractions greater than one. Then multiply and write the result as a mixed number, if possible.

Example:

$2\frac{1}{2} \cdot 1\frac{1}{4}$

$\frac{5}{2} \cdot \frac{5}{4}$

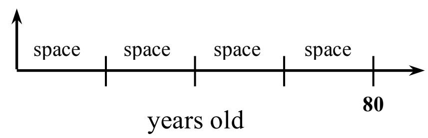
$\frac{25}{8}$

$3\frac{1}{8}$

Intervals and Scaling

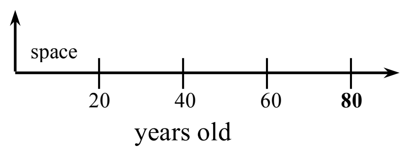
The numbers on the axes of a graph show the **scaling** of the axes. The difference between consecutive markings tells the size of the **interval**. When you scale each axis, you must use equal intervals to represent the data accurately. For example, an interval of 5 creates a scale numbered 0, 5, 10, 15, etc. Unequal intervals distort the relationship in the data. It is important to note that horizontal and vertical axes *do not* need the same scaling. In fact, it is often convenient to choose different scales for the two axes.

Notice on the graph at right that 80 marks the end of the *fourth* interval. If you divide 80 years by 4, you can see that the length of an interval on this graph is 20.



$(80 \div 4 = 20)$

The second graph at right has each interval labeled. This is called "scaling the axis."



Notes: